Name _____ MTH 1001 - Calculus 1

PRACTICE ADVANCED STANDING EXAM

1. (a) Write the general *definition of the derivative* for a function $f(x)$
[14pts]

(b) Find $f'(x)$ by using the *definition of the derivative* with the following function:
$f(x) = \frac{1}{x}$

2. Find the derivative: $f(x) = x^3 \tan(2x-1)$
[12pts]

3. Find the derivative: $f(x) = e^{x^3} + \ln(\sec x) + \csc(\ln x)$
[9pts]

4. Find the derivative $\frac{dy}{dx}$ for the following: $x^2 + y^3 = ye^{5x}$

[10pts]

5. Find the derivative: $f(x) = \frac{\sin x}{x} + \sin^{-1} x + \sinh x$

[9pts]

6. Find the derivative: $y = x^{3x}$

[12 pts]

7. [10pts] A toy car moves along a straight track during time $0 \le t \le 4$. It's position at any time from a fixed point along the track is given by $s(t) = t^3 - 3t^2$
Answer the following about the motion of the car.
(Note: The time *t* is measured in minutes and distance *s* in inches.)

(a) What is the position, velocity, and acceleration of the car at the time *t* = 3 minutes?

(b) At what time does the car come to a stop?

8. [12pts] A 5 ft ladder is leaning against a wall and starts to slide. How fast is the bottom edge of the ladder moving along the floor when the top corner of the ladder is 3 ft up the wall and sliding down the wall at a rate of 8 ft/sec?

9. [8pts] Use L'Hôpital's Rule to evaluate the following limit:

$$\lim_{x \to 0} \frac{x^3 + 5\sin x}{x\cos x}$$

10. Graph the following Rational Function:

$$f(x) = \frac{36(x-1)}{x^2}$$

[16 pts]

Hint: $f'(x) = \frac{36(2-x)}{x^3}$ and $f''(x) = \frac{72(x-3)}{x^4}$

(Use calculus to find the locations of any important points [maxs, mins, pts of inflection] and label them on the graph.)

11. [12 pts] A box with a closed top is going to be manufactured so that its base is a square and its volume will be $100\,\text{cm}^3$. If the material to make the top and bottom of the box cost \$50 per square cm and the material for the sides costs \$4 per square cm, find the dimensions that will minimize the cost of the box.

12. [16pts] Find the exact area under the curve $f(x) = 2x + 1$ over the interval $[a,b]$, where x_i is the right endpoint of each equal subinterval, given $a = 1$ and $b = 3$.

Hint – Evaluate the limit: $\lim_{n\to+\infty} \sum_{i=1}^{n} f(x_i)\Delta x$

$\sum_{i=1}^{n}(1) = n$ $\quad$ $\sum_{i=1}^{n}(i) = \frac{n(n+1)}{2}$ $\quad$ $\sum_{i=1}^{n}(i^2) = \frac{n(n+1)(2n+1)}{6}$ $\quad$ $\sum_{i=1}^{n}(i^3) = \left[\frac{n(n+1)}{2}\right]^2$

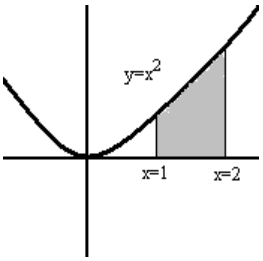
13. Evaluate the indefinite integral: $\int \left(50x^4 + 10x^3 + 12\sqrt{x}\right)dx$

[12pts]

Setup a definite integral and find the area of the indicated regions:

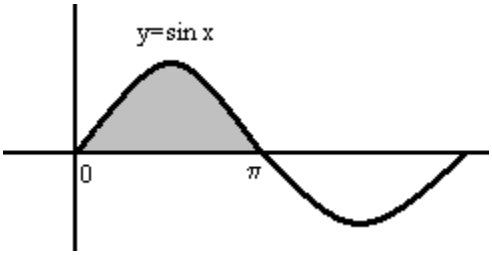
14.

[5pts]



15.

[5pts]



16. Evaluate the following: $\frac{d}{dx}\left(\int_4^{x^3} e^{T^2}\,dT\right)$

[10pts]

17. Evaluate the indefinite integral: $\int \left[24\sin^2(4x)\cos(4x)\right]dx$

[14pts]

18. Evaluate the definite integral: $\int_0^1 \left[8x\left(x^2+1\right)^3\right]dx$

[14pts]